A Comparison of Three Closed-Loop PID Tuning Algorithms

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The PID controller remains the most widely used controller in the process industries. Tuning of these controllers usually follows two steps. The first involves using one of several "tuning methods" to first obtain a simple approximation to the process dynamics. This information is then used to calculate appropriate tuning parameters. In practice these parameters serve as good first guesses and are then modified in a second (trial and error) stage called "fine tuning."

Yuwana and Seborg (1982) introduced a popular tuning method (YS) which had great practical advantage since it required only a single closed-loop test on the process. The YS method avoided the disadvantages of the Zeigler Nichols Method (1942), which required continuous plant cycling, and the Cohen-Coon (1953) method, which requires an open-loop test on the process and is thus riskier.

One of the difficulties with the YS method was its inability to provide suitable tuning for process with large time delay. This was due to their use of a first-order Padé approximation for the process model time delay. They also used an additional approximation to calculate the deadtime and time constant of the process model.

Jutan and Rodriguez (1984) (JR) modified the YS algorithm by using a higher-order approximation for the process model time delay. They used a least squares method to optimize the parameters in their approximation, which also provided for an analytical solution to the closed-loop dynamics.

Simulation studies using the JR method showed that this modification significantly improved tuning over the YS method, for processes with large deadtimes.

Lee (1989) introduced a modification to the YS algorithm which avoids approximating the process model time delay term. He instead matched the dominant poles of the closed-loop system with a second order process response in order to determine the process parameters.

This note evaluates Lee's method and compares it to the two previous tuning methods.

Theoretical Development

Figure 1 shows a typical closed-loop system. If the controller G_C is made proportional only, i.e., $G_c = K_c$, and we assume the process (Gp) is well approximated by a first-order plus dead-time model,

$$G_m = \frac{K_m e^{-d_m s}}{\tau_m s + 1} \tag{1}$$

then the closed-loop transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{K_c K_m e^{-d_m s}}{1 + \tau_m s + K_c K_m e^{-d_m s}} \tag{2}$$

A step change in set point, R, produces the typical response in Figure 1b. In order to obtain an analytical solution to Eq. 2 Yuwana and Seborg (1982) approximated the delay term $e^{-d_m s}$ in the denominator by a first-order Padé form,

$$e^{-d_m s} = \frac{1 - 0.5\, d_m s}{1 + 0.5\, d_m s}$$

whereas Jutan and Rodriguez (1984) used

$$e^{-d_m s} = \frac{1 + \gamma_1 d_m s + \gamma_2 d_m{}^2 s^2}{1 + \delta d_m s}$$

Both of these approaches lead to a second-order approximation to Eq. 2 as

$$\frac{C(s)}{R(s)} \approx \frac{K'(1 + qs)}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{3}$$

where $K' = K_c K_m$.

Using the solution to Eq. 3, extreme points Cp_1, Cp_2, Cm_1 on

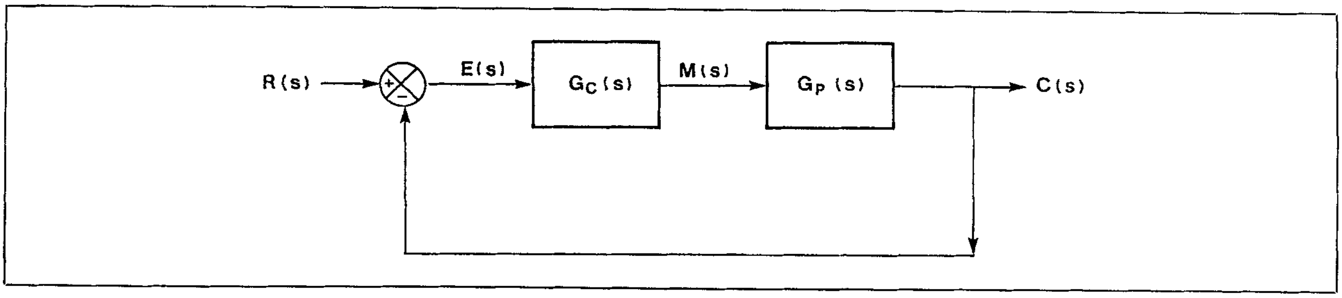

Figure 1a. Closed-loop feedback system.

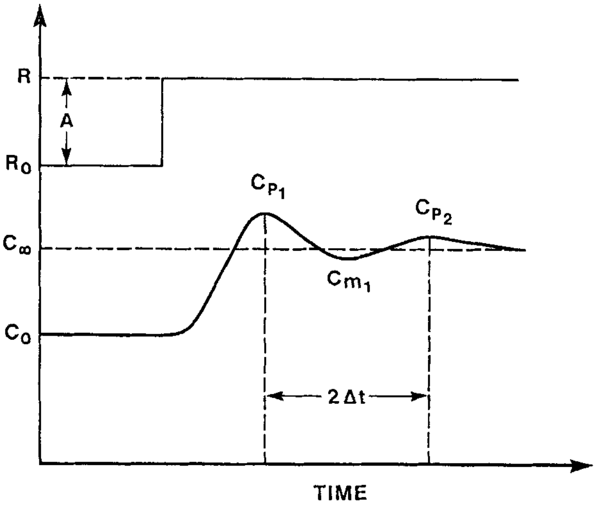

Figure 1b. Closed-loop step test.

Figure 1b can be used to estimate the process parameters in Eq. 1 using both the YS and JR algorithms.

Lee (1989) argued the problem in reverse. His solution is based on the observation that Figure 1b is typical of an underdamped, closed-loop, process response to a step change in setpoint with $G_c = K_c$. He further claimed that Eq. 3, *irrespective of how it is derived*, is of a suitable form to match the response in Figure 1b. He then matched the dominant poles of transfer function (Eq. 2) to the poles of transfer function (Eq. 3) in order to obtain an estimate of the process parameters in Eq. 1. This required the solution to a set of nonlinear equations,

$$e^{-\alpha d_m} - \frac{K'}{\beta}[\alpha \sin(\beta d_m) - \beta \cos(\beta d_m)] = 0 \tag{4}$$

$$\tau_m = \frac{1}{\alpha}[1 + K' e^{\alpha d_m} \cos(\beta\, d_m)] \tag{5}$$

where $\alpha = \zeta/\tau$ and $\beta = (\sqrt{1 - \zeta^2})/\tau$.

Lee (1989) provides a simple iterative method for the solution to Eq. 4 and hence Eq. 5, which converged rapidly in all the examples we examined.

Choice of test K_c

Jutan and Rodriguez (1984) showed that both YS method and the JR method produce poor tuning if the choice $G_c = K_c$, for the step test in Figure 1b, was too low. This is illustrated in Figure 2. Peak point data obtained from the less oscillatory curve in Figure 2 (perhaps because it is a poorer match to Eq. 3) generally gave poor model parameter estimates and poor tuning, (see Table 1).

This sensitivity to low K_c observed by Jutan and Rodriguez (1984) is important, since, in practice, a suitable K_c is not usually known in advance. Jutan and Rodriguez (1984) found that a suitable value of the test K_c was a value close to the optimal K_c for a PID controller after the tuning process was complete. Thus some trial and error in the choice of the test K_c was required.

It should also be understood that all three methods (YS, JR, and Lee) provide, at best, an approximate model fit, since only a minimal number of data points along the response curve are used to estimate the model parameters. It is therefore important that these methods should be relatively insensitive to the exact value of the peak data (Cp_1, C_{m_1}, Cp_2) and its location in time.

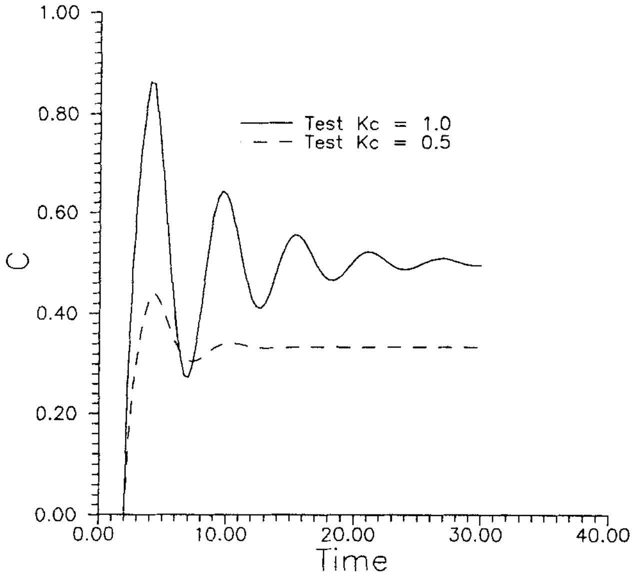

Figure 2. Tuning test: effects of K_c.

Table 1. Comparison of Tuning Methods

Test	Process			YS			JR			Lee		
K_c	K_p	τ_p	d_p	K_m	τ_m	d_m	K_m	τ_m	d_m	K_m	τ_m	d_m
0.5	1	1	2	1	1.43	1.62	1	0.83	2.45	1	1.02	1.96
1.0	1	1	2	1	1.54	2.02	1	1.05	2.26	1	1.0	1.97
0.75	1	1	4	0.99	2.11	3.7	0.99	1.06	4.65	0.99	1.1	3.87

Table 2. Tuning Comparison

	Model			Tuning (PID)		
	K_m	τ_m	d_m	K_c	T_i	T_d
YS	1.01	4.04	4.76	1.15	5.37	1.81
JR	1.01	2.88	5.37	0.75	5.37	2.03
Lee	1.01	2.84	4.68	0.84	4.83	1.77

Simulations

The same examples considered by Lee (1989) were resimulated for comparison. Data are presented only for large dead times since for small dead times all three methods produce suitable tuning.

Example 1

$$G_p(s) = \frac{e^{-d_p s}}{s + 1}$$

This process was simulated and tuned using two-step test values for $K_c = (0.5\ 1.0)$ for $d_p = 2$, as shown in Figure 2, and $K_c = 0.75$ for $d_p = 4$.

The results in Table 1 show that while model parameters are closer to the process parameters at test $K_c = 1.0$, for all three methods, Lee's method is quite insensitive to the test K_c used. The JR method gives an adequate process model at $K_c = 1.0$ whereas YS method is unsatisfactory at both levels of K_c. Lee's method provides a consistently good process model.

For very large dead time ($d_p = 4$) both the JR and Lee's method provide adequate models whereas the YS method has increasing difficulty especially in estimating the time constant τ_m.

Example 2

This example follows Lee (1989) and compares tuning for a high-order process with large dead time.

$$G_p(s) = \frac{e^{-3s}}{(s + 1)^2(2s + 1)}$$

The three tuning methods were used to tune this process using $K_c = 1.0$ (a suitable value). The tuning is shown in Table 2 and used the ITAE method (Miller et al., 1967) for Load disturbance. Table 2 shows the approximating model and calculated tuning parameters.

Consistent with Lee's (1989) and Jutan and Rodriguez's (1984) analysis, the YS method gives rise to controller gain which is too high and thus leads to a response that is too oscillatory.

Figure 3 shows the responses of the three methods to a unit step in load. Lee's method produces the best tuning; the JR method is satisfactory, while the YS method is too oscillatory.

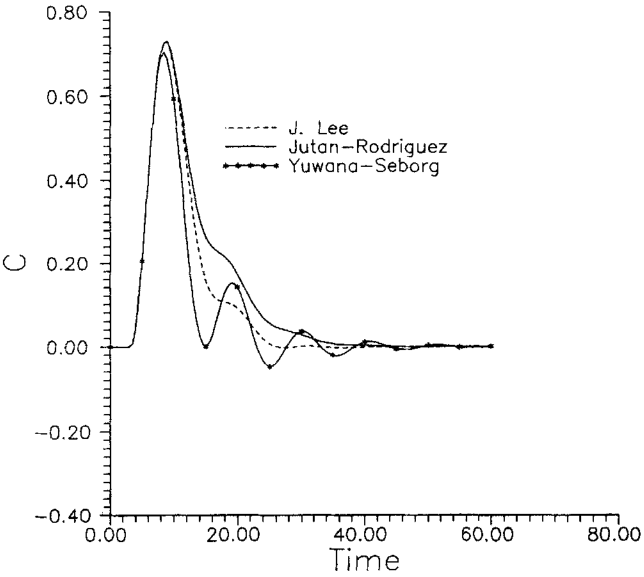

Figure 3. Tuning comparison: load change.

Notation

A = step size of set point
$C(s)$ = process response variable
Cm_1 = first minimum of process response
Cp_1 = first maximum of process response
Cp_2 = second maximum of process response
C_o = initial processes response
C_∞ = final processes response
$d_{m,p}$ = model, process dead time
$G_{c,m,p}$ = controller, model, process, transfer function
$K_{c,m,p}$ = controller, model, process, gain
q = parameter
R_o = initial set point
R = final set point
s = Laplace variable
t = time

Greek letters

τ = time constant
ζ = damping coefficient
α = derived parameters in Eqs. 4 and 5
β = derived parameters in Eqs. 4 and 5
$\gamma_1, \gamma_2, \delta$ = fitted coefficients following Eq. 2
Δt = half period of oscillation

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